Generalized Slip Condition

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IRREGULAR SURFACES, an equivalent description

**Goal**
A computationally cheap BC to simulate fluids flowing over generic rough walls

Rough surface under investigation, composed of a hexagonal periodic lattice:
- \( \text{FS microscopic cell} \ (l_1 \times l_2 \times l_3 \simeq l) \)
- porosity \( \vartheta = |F|/|\text{FS}| \)
- \( \text{ES fictitious equivalent surface} \)
- \( \epsilon = \frac{l}{L} \ll 1 \)
THE METHOD based on homogenization

Further hypotheses:

- up to
  \[ \text{Re} = U^{out} L/\nu = \mathcal{O}(1/\epsilon) \]
- every kind of microscopic structure

After some algebra...

\[ \text{Re} \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \nabla^2 u_i \]

\[ \frac{\partial u_i}{\partial x_i} = 0 \]
THE RESULTING MODEL

\[ u_i = -\epsilon^2 \text{Re} \left( K_{ij} \frac{\partial p}{\partial x_j} + \epsilon L_{ilk} \left( \frac{\partial u_l}{\partial x_k} + \frac{\partial u_k}{\partial x_l} \right) \right) |_{x \in ES}, \]

\[ K_{ij} = \langle K_{ij} \rangle, \quad L_{ijk} = \langle L_{ijk} \rangle, \quad A_j = \langle A_j \rangle, \quad B_{lk} = \langle B_{lk} \rangle, \quad \langle f \rangle := \frac{1}{|FS|} \int_F f \, dV, \]

THE RESULTING MODEL

\[ u_i = -\epsilon^2 \text{Re} \, \mathcal{K}_{ij} \frac{\partial p}{\partial x_j} + \epsilon \, \mathcal{L}_{ilk} \left( \frac{\partial u_l}{\partial x_k} + \frac{\partial u_k}{\partial x_l} \right) \bigg|_{x \in \mathbb{E}S}, \]

\[ \mathcal{K}_{ij} = \langle K_{ij} \rangle, \quad \mathcal{L}_{ijk} = \langle L_{ijk} \rangle, \quad A_j = \langle A_j \rangle, \quad B_{lk} = \langle B_{lk} \rangle, \quad \langle f \rangle := \frac{1}{|\mathbb{F}S|} \int_{\mathbb{F}} f \, dV, \]

\[
\begin{cases}
\frac{\partial A_j}{\partial x_i} - \nabla^2 K_{ij} = \delta_{ij}, \\
\frac{\partial K_{ij}}{\partial x_i} = 0 \\
K_{ij} = 0 \quad \text{on} \quad \partial \mathbb{S}, \\
\frac{\partial K_{jl}}{\partial x_3} + \frac{\partial K_{3l}}{\partial x_j} = 0, \quad x_3 \in \mathbb{T}
\end{cases}
\]

\[
\begin{cases}
- \frac{\partial B_{j3}}{\partial x_i} + \nabla^2 L_{ij3} = 0, \\
\frac{\partial L_{ij3}}{\partial x_i} = 0, \\
L_{ij3} = 0 \quad \text{on} \quad \partial \mathbb{S}, \\
\frac{\partial L_{pj3}}{\partial x_3} + \frac{\partial L_{3j3}}{\partial x_p} = \delta_{jp}, \quad x_3 \in \mathbb{T},
\end{cases}
\]

\[ j = 1, 2, \forall \, i, \, K_{ij}, \, A_j, \, L_{ij3} \text{ and } B_{j3} \] \( x_1, x_2 \) periodic.

MICROSCOPIC PROBLEMS: the surface permeability $\mathcal{K}_{ij}$

\[
\begin{align*}
\frac{\partial A_{ij}}{\partial x_i} - \nabla^2 K_{ij} &= \delta_{ij} \\
\frac{\partial K_{ij}}{\partial x_i} &= 0 \\
K_{ij} &= 0 \text{ on } \partial S \\
\frac{\partial K_{jl}}{\partial x_3} + \frac{\partial K_{3l}}{\partial x_j} &= 0, \ x_3 \in \mathbb{T} \\
K_{ij} &= \text{ } x_1, x_2\text{-periodic}
\end{align*}
\]

$\mathcal{K}_{ij} = 0 \ i \neq j$ by symmetry
$\mathcal{K}_{33} = 0$ by axis orientation

weak anisotropy: $\mathcal{K}_{11} \neq \mathcal{K}_{22}$
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MICROSCOPIC PROBLEMS: the slip tensor $\mathcal{L}_{ijk}$

\[
\begin{align*}
- \frac{\partial B_{l3}}{\partial x_i} + \nabla^2 L_{il3} &= 0 \\
\frac{\partial L_{il3}}{\partial x_i} &= 0 \\
L_{il3} &= 0 \text{ on } \partial S, \\
\frac{\partial L_{pl3}}{\partial x_3} + \frac{\partial L_{3l3}}{\partial x_p} &= \delta_{lp}, x_3 \in \mathbb{T} \\
L_{il3} &= x_1, x_2\text{-periodic}
\end{align*}
\]

$\mathcal{L}_{ii3} \neq 0 \quad i = 1, 2$ by symmetry and axis orientation

weak anisotropy: $\mathcal{L}_{113} \neq \mathcal{L}_{223}$
MICROSCOPIC PROBLEMS: the slip tensor $L_{ijk}$

\[
\begin{aligned}
&- \frac{\partial B_{l3}}{\partial x_i} + \nabla^2 L_{il3} = 0 \\
&\frac{\partial L_{il3}}{\partial x_i} = 0 \\
&L_{il3} = 0 \quad \text{on } \partial S, \\
&\frac{\partial L_{pl3}}{\partial x_3} + \frac{\partial L_{3l3}}{\partial x_p} = \delta_{lp}, \quad x_3 \in \mathbb{T} \\
&L_{il3} \quad x_1, x_2\text{-periodic}
\end{aligned}
\]

$L_{ii3} \neq 0 \quad i = 1, 2$ by symmetry and axis orientation

weak anisotropy: $L_{113} \neq L_{223}$
VALIDATION: flows past rough spherical particles

Free uniform unidirectional flow \((U^{\text{out}}, 0, 0)\) past a spherical particle such that:

- \(\vartheta = 0.60\)
- 1440 protrusions, i.e. \(\epsilon \approx 0.03\)
- \(\text{Re} = 100\)
- \(200R \times 80R \times 80R\)
- 15 to 25 millions cells
- OpenFOAM in parallel with up to 500 cores
- up to \(1.2 \times 10^5\) CPU hours for each DNS to reach the steady state at \(\text{Re}= 100\)
VALIDATION: local comparisons

![Graphs showing validation results](attachment:validation_graphs.png)
VALIDATION: local comparisons
A robust BC for incompressible fluid flows above rough (and smooth?) surfaces has been developed and validated in a non trivial case (showing a good agreement also when the homogeneity is weakened).

- go further in the $\epsilon$-expansion to develop a strategy to better locate the equivalent surface in the macroscopic analogy
- extend the procedure in a stochastic direction to analyze irregular surfaces without a fixed shape of the protrusions
- study the dynamics of Janus-like spheres optimizing distribution and shape of the protrusions to enhance their dynamic performances
- following the path of drag reduction, homogenization of two phase flows is necessary