Effective stress jump condition across microstructured thin membranes

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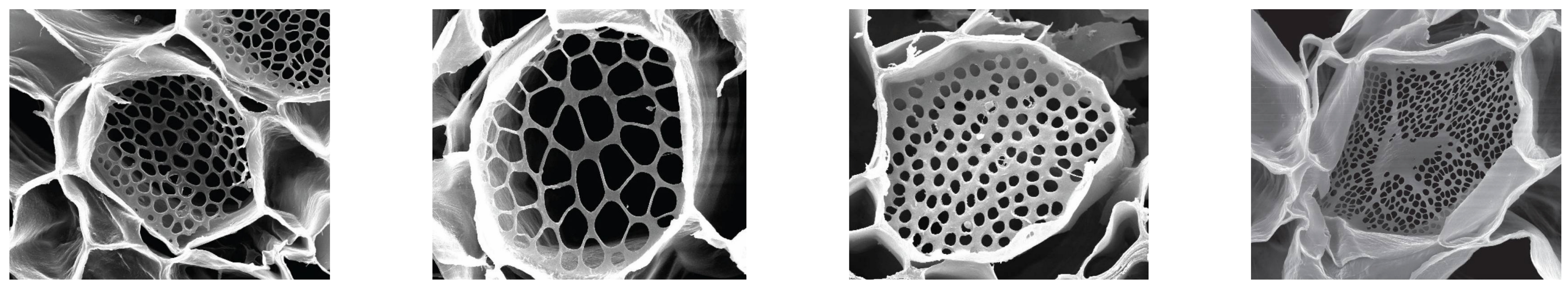
Introduction

A Darcy like model is often chosen to simulate the flow of an incompressible fluid across thin microstructured porous membranes.



Can we do better with a more formal approach?

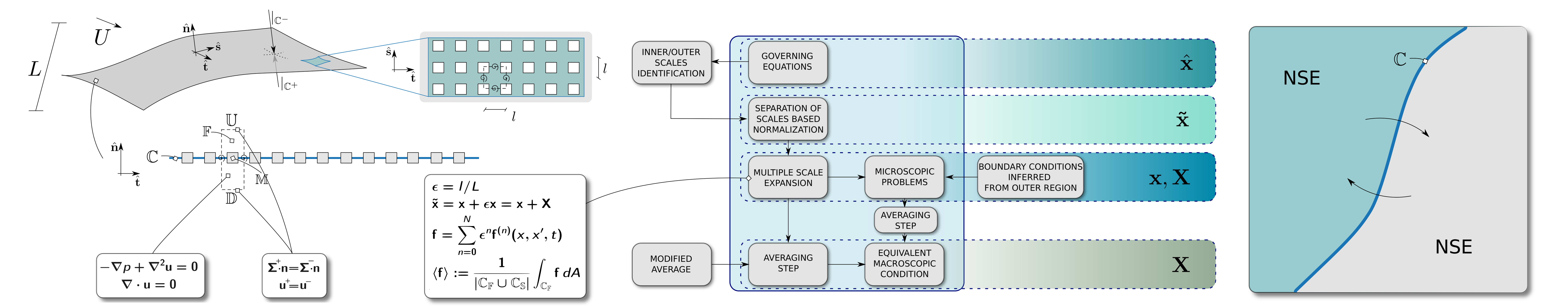
References

Zampogna, G. A. & Gallaire, F. 2019 Effective stress jump across membranes. *JFM*, under review.

Objectives

1. A theoretical model for flows through microstructured thin porous membranes.
2. A numerical implementation of the model.

The method: a homogenization technique



Effective stress jump condition

Homogenization is used to obtain the effective interface condition. The unknowns, $\mathbf{u}$ and p, are the leading order approximation of the velocity and pressure fields and vary only over the macroscale. The interface conditions to be imposed on the homogeneous domain $\mathbb{C}$ are:

$$\begin{cases} \mathbf{u}|_{\mathbb{C}} = \boldsymbol{\mathcal{M}} : \boldsymbol{\Sigma}|_{\mathbb{C}^-} + \boldsymbol{\mathcal{N}} : \boldsymbol{\Sigma}|_{\mathbb{C}^+} \\ \mathbf{u}|_{\mathbb{C}^-} = \mathbf{u}|_{\mathbb{C}} = \mathbf{u}|_{\mathbb{C}^+} \\ \boldsymbol{\Sigma}|_{\mathbb{C}^-} = -\alpha \mathbf{I} p|_{\mathbb{C}^-} + \beta(\nabla \mathbf{u} + \nabla \mathbf{u}^T)|_{\mathbb{C}^-} \quad \alpha, \beta \in \{1, \epsilon\} \\ \boldsymbol{\Sigma}|_{\mathbb{C}^+} = -\gamma \mathbf{I} p|_{\mathbb{C}^+} + \delta(\nabla \mathbf{u} + \nabla \mathbf{u}^T)|_{\mathbb{C}^+} \quad \gamma, \delta \in \{1, \epsilon\} \end{cases}$$

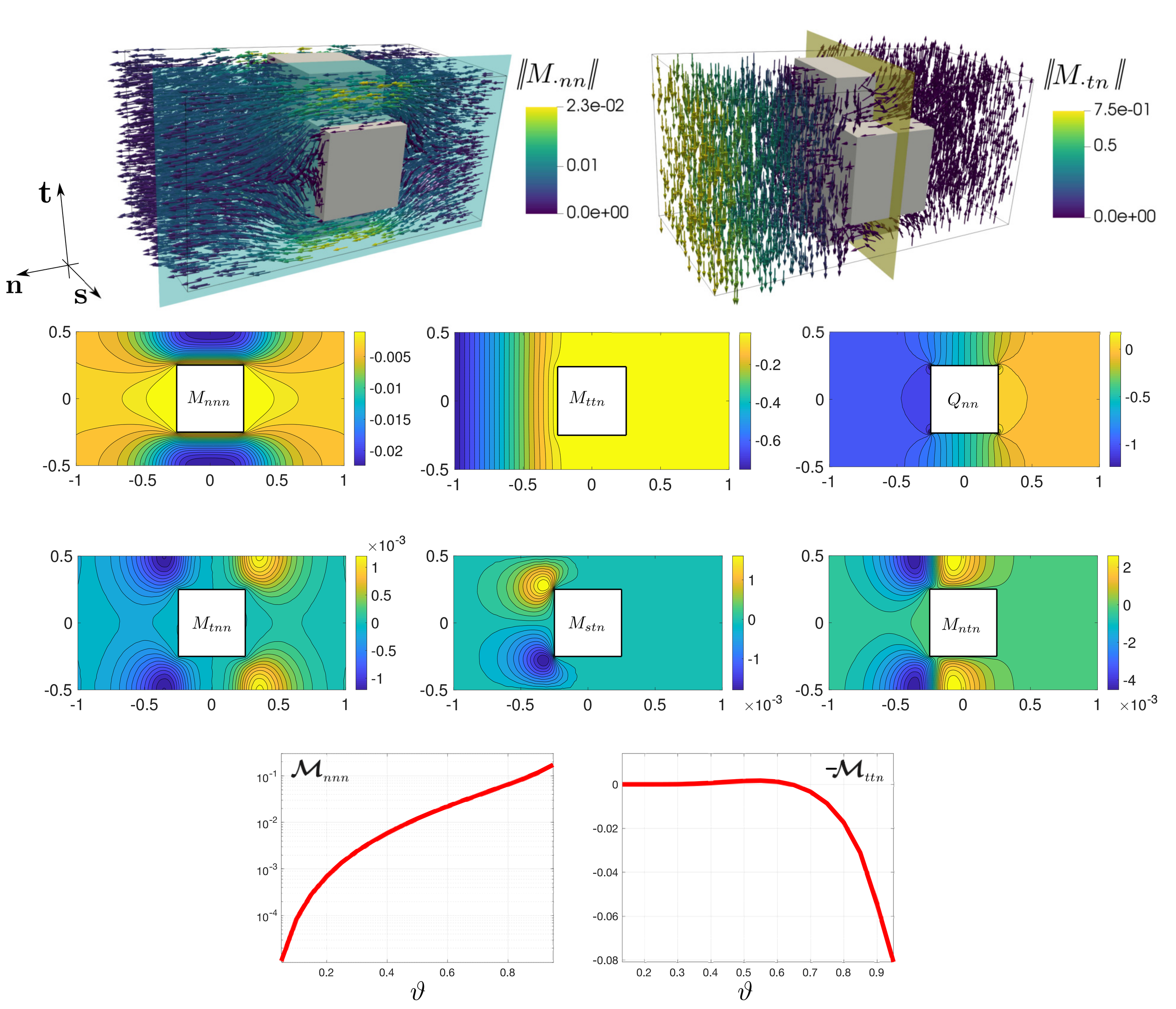
where $\boldsymbol{\mathcal{M}}$ and $\boldsymbol{\mathcal{N}}$ are averages of microscopic third order tensors unknown.

Numerical tools

- FEM solver for microscopic problems.
- FEM + DDM solver for macroscopic flow.
- DNS for validation purposes solved with FEM.

Microscopic solution of the effective tensors

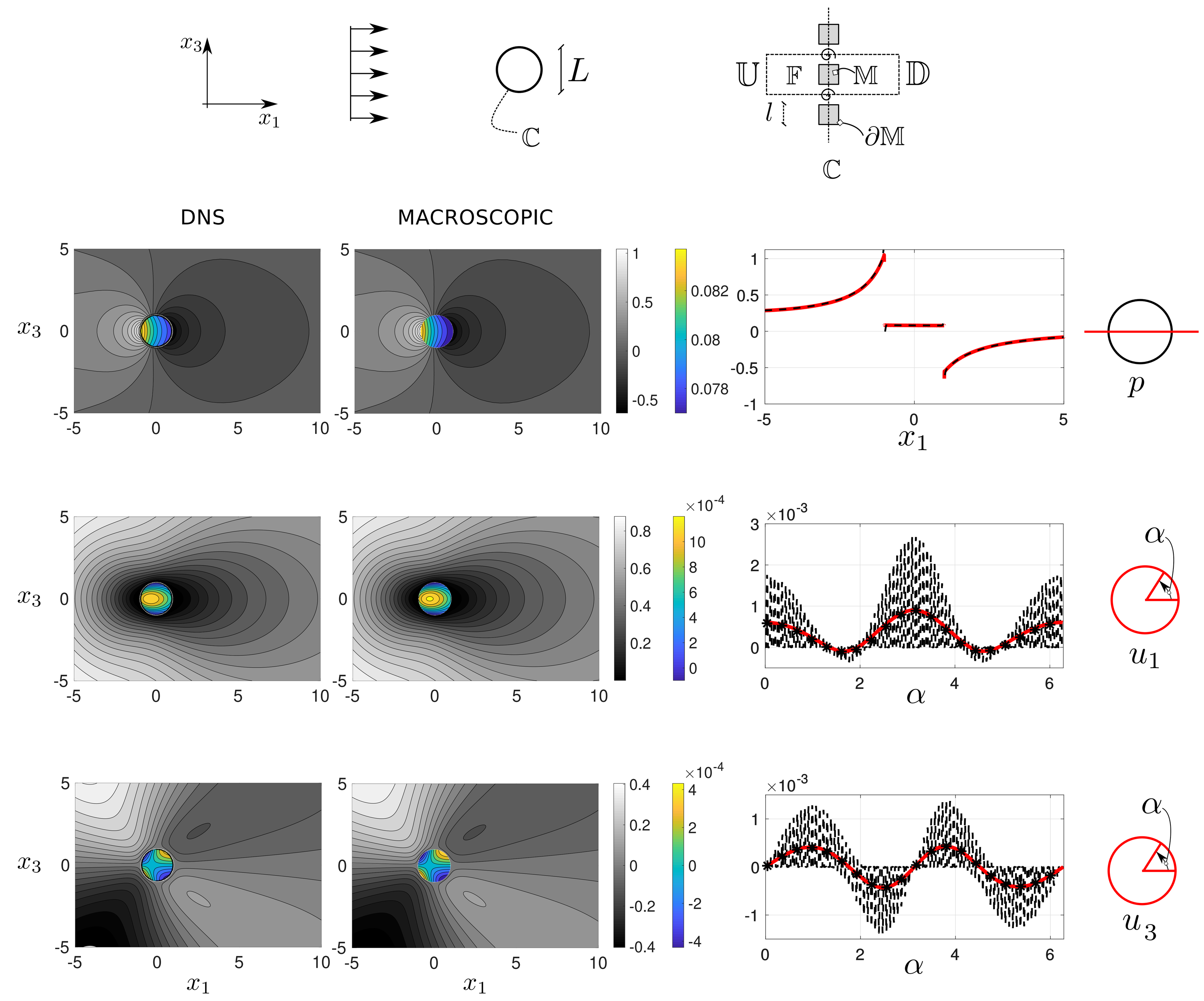
- the upward Navier tensor $\boldsymbol{\mathcal{M}} = \langle \mathbf{M} \rangle$, with $\mathbf{M}$ such that $\{\nabla \mathbf{Q} - \nabla^2 \mathbf{M} = \mathbf{0}, \nabla \cdot \mathbf{M} = \mathbf{0}\}$ on $\mathbb{F}$, $\mathbf{M} = \mathbf{0}$ on $\partial|_{\mathbb{FS}}$, $\boldsymbol{\Sigma}(\mathbf{Q}, \mathbf{M}) = \mathbf{I}$ on $\mathbb{U}$ and $\boldsymbol{\Sigma}(\mathbf{Q}, \mathbf{M}) = \mathbf{0}$ on $\mathbb{D}$



- the downward Navier tensor $\boldsymbol{\mathcal{N}} = \langle \mathbf{N} \rangle$ is deduced from $\boldsymbol{\mathcal{M}}$ via geometrical considerations

Macroscopic solution of the effective flow field

- validation case: free uniform flow past a cylindrical microstructured shell.



Conclusion & Perspectives

- We developed and validated an effective macroscopic model describing the flow across thin microstructured porous membranes.
- The procedure has to be iterated to address the final goal of the project, i.e. providing an effective description of hierarchical membranes.

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