Generalized Slip Condition

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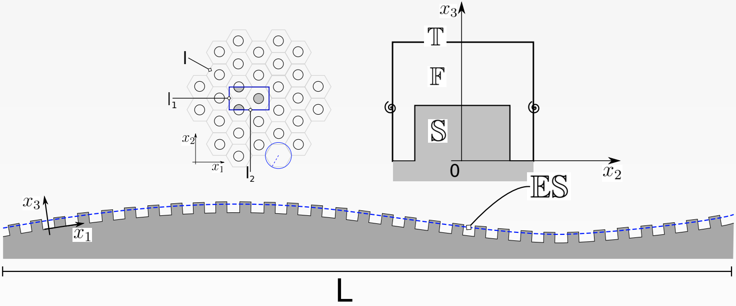
IRREGULAR SURFACES, an equivalent description

Goal

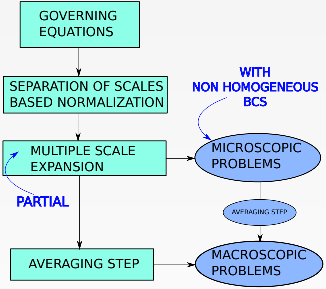
A computationally cheap BC to simulate fluids flowing over generic rough walls

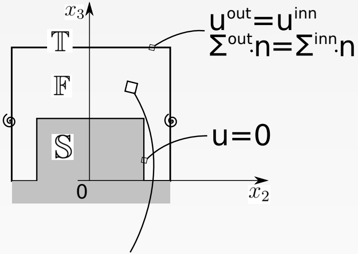


Rough surface under investigation, composed of a hexagonal periodic lattice:

- $\mathbb{FS}$ microscopic cell ($l_1 \times l_2 \times l_3 \simeq l$)
- porosity $\vartheta = |\mathbb{F}|/|\mathbb{FS}|$
- $\mathbb{ES}$ fictitious equivalent surface
- $\epsilon = \frac{l}{L} \ll 1$

THE METHOD based on homogenization





$$\text{Re}\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=-\frac{\partial p}{\partial x_i}+\nabla^2 u_i$$

$$\frac{\partial u_i}{\partial x_i}=0$$

Further hypotheses:

- up to $\text{Re} = U^{out}L/\nu = \mathcal{O}(1/\epsilon)$
- every kind of microscopic structure

After some algebra...

THE RESULTING MODEL

$$u_i = -\epsilon^2 \mathrm{Re}\, \mathcal{K}_{ij} \frac{\partial p}{\partial x_j} + \epsilon\, \mathcal{L}_{ilk} \left(\frac{\partial u_l}{\partial x_k} + \frac{\partial u_k}{\partial x_l} \right) \Big|_{\mathbf{x} \in \mathbb{ES}},$$

$$\mathcal{K}_{ij} = \langle K_{ij} \rangle,\ \mathcal{L}_{ijk} = \langle L_{ijk} \rangle,\ \mathcal{A}_j = \langle A_j \rangle,\ \mathcal{B}_{lk} = \langle B_{lk} \rangle,\ \langle f \rangle := \frac{1}{|\mathbb{FS}|} \int_{\mathbb{F}} f\, dV,$$

cf. Beavers & Joseph (1967) and Navier (1823).

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$$\begin{cases} \dfrac{\partial A_j}{\partial x_i} - \nabla^2 K_{ij} = \delta_{ij}, \\ \dfrac{\partial K_{ij}}{\partial x_i} = 0 \\ K_{ij} = 0 \quad \text{on } \partial\mathbb{S}, \\ \dfrac{\partial K_{jl}}{\partial x_3} + \dfrac{\partial K_{3l}}{\partial x_j} = 0, x_3 \in \mathbb{T} \end{cases}$$

$$\begin{cases} -\dfrac{\partial B_{j3}}{\partial x_i} + \nabla^2 L_{ij3} = 0, \\ \dfrac{\partial L_{ij3}}{\partial x_i} = 0, \\ L_{ij3} = 0 \quad \text{on } \partial\mathbb{S}, \\ \dfrac{\partial L_{pj3}}{\partial x_3} + \dfrac{\partial L_{3j3}}{\partial x_p} = \delta_{jp}, x_3 \in \mathbb{T}, \end{cases}$$

$j = 1, 2,\ \forall\ i,\ K_{ij},\ A_j,\ L_{ij3}$ and B_{j3} x_1, x_2 periodic.

cf. Beavers & Joseph (1967) and Navier (1823).

MICROSCOPIC PROBLEMS: the surface permeability $\mathcal{K}_{ij}$

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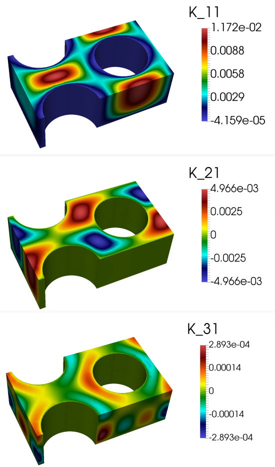

$\mathcal{K}_{ij} = 0 \quad i \neq j$ by symmetry
$\mathcal{K}_{33} = 0$ by axis orientation

weak anisotropy: $\mathcal{K}_{11} \neq \mathcal{K}_{22}$

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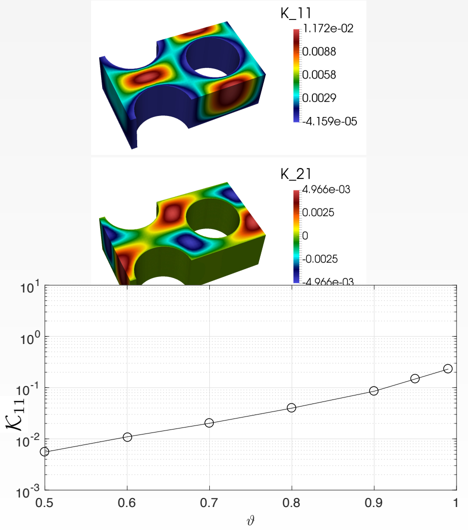
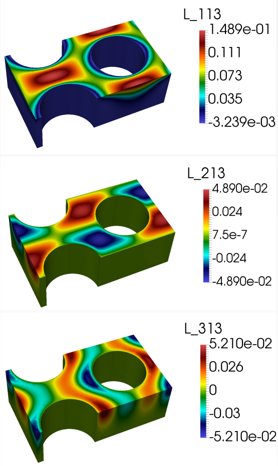


MICROSCOPIC PROBLEMS: the slip tensor $\mathcal{L}_{ijk}$

$$\begin{cases} -\dfrac{\partial B_{l3}}{\partial x_i} + \nabla^2 L_{il3} = 0 \\ \dfrac{\partial L_{il3}}{\partial x_i} = 0 \\ L_{il3} = 0 \quad \text{on } \partial\mathbb{S}, \\ \dfrac{\partial L_{pl3}}{\partial x_3} + \dfrac{\partial L_{3l3}}{\partial x_p} = \delta_{lp}, x_3 \in \mathbb{T} \\ L_{il3} \quad x_1, x_2\text{-periodic} \end{cases}$$

$\mathcal{L}_{ii3} \neq 0 \quad i = 1, 2$ by symmetry and axis orientation

weak anisotropy: $\mathcal{L}_{113} \neq \mathcal{L}_{223}$



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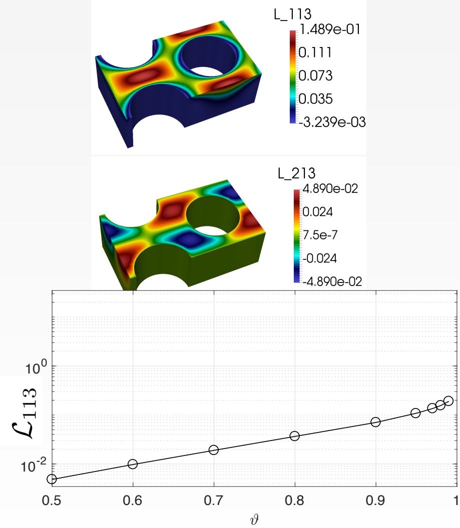
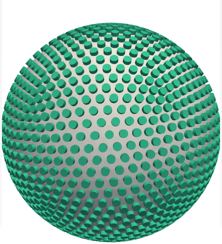
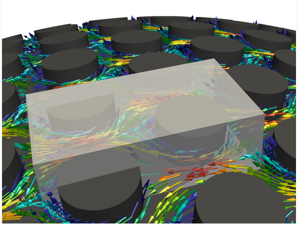


VALIDATION: flows past rough spherical particles

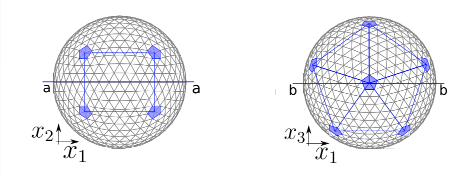
Free uniform unidirectional flow $(U^{out}, 0, 0)$ past a spherical particle such that:

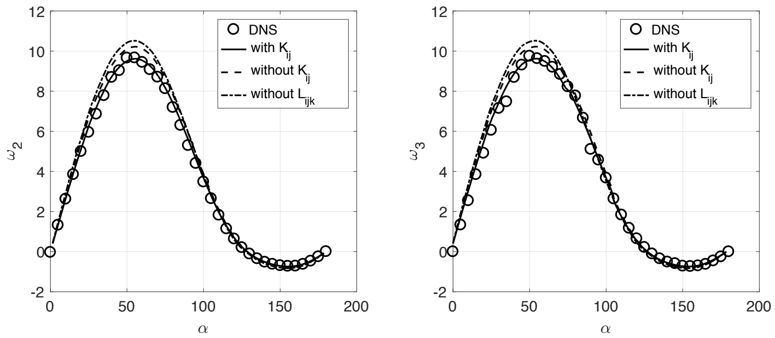
- $\vartheta = 0.60$
- 1440 protrusions, i.e. $\epsilon \approx 0.03$
- Re $= 100$
- $200R \times 80R \times 80R$
- 15 to 25 millions cells
- OpenFOAM in parallel with up to 500 cores
- up to 1.2×10^5 CPU hours for each DNS to reach the steady state at Re$= 100$





VALIDATION: local comparisons





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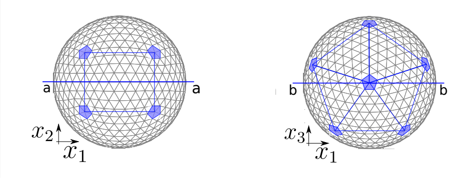

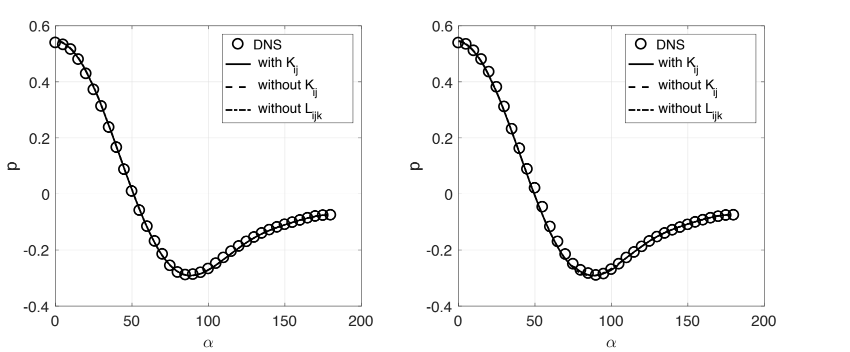

CONCLUSIONS and more

A robust BC for incompressible fluid flows above rough (and smooth?) surfaces has been developed and validated in a non trivial case (showing a good agreement also when the homogeneity is weakened)

- go further in the ϵ-expansion to develop a strategy to better locate the equivalent surface in the macroscopic analogy
- extend the procedure in a stochastic direction to analyze irregular surfaces without a fixed shape of the protrusions
- study the dynamics of Janus-like spheres optimizing distribution and shape of the protrusions to enhance their dynamic performances
- following the path of drag reduction, homogenization of two phase flows is necessary